04/06/2024

QUANTUM FLUIDS OF LIGHT SUMMER SCHOOL 2024

Polarization dynamics of trapped polariton condensates with PT-symmetry

I. Jesán Velázquez-Reséndiz and Yuri G. Rubo
Phys. Rev. B **109**, 085312

I. Jesán Velázquez-Reséndiz
vereij@ier.unam.mx

IER Instituto de Energías Renovables

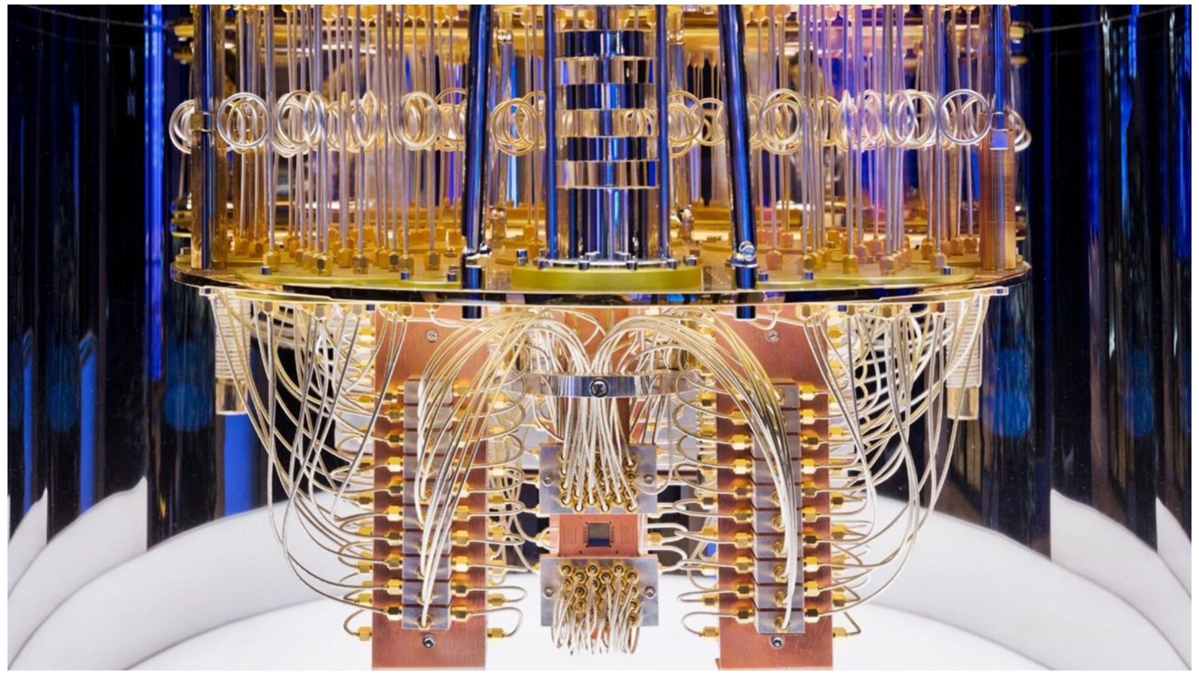
CONTENTS

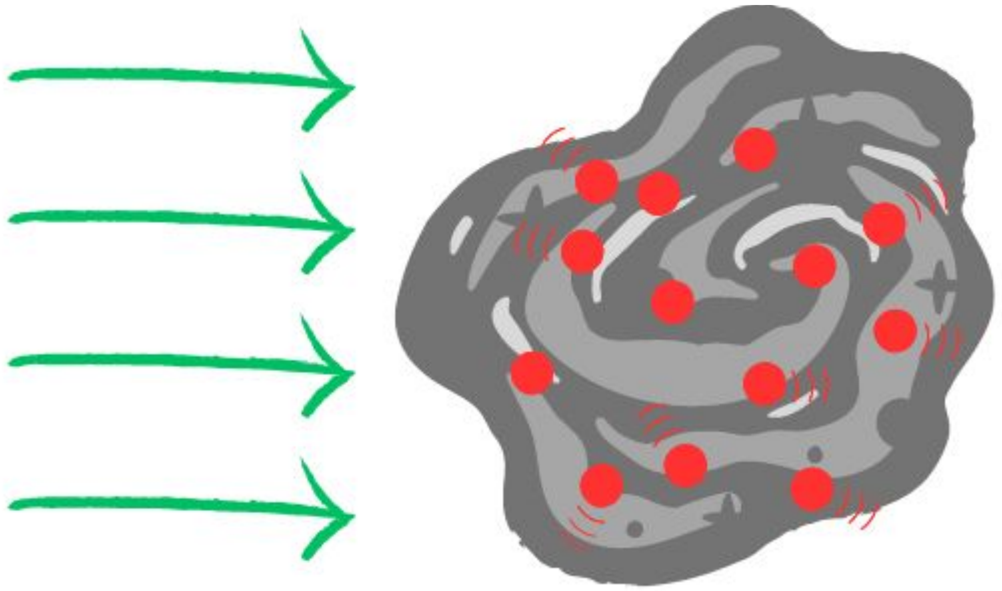
01

Introduction

Why?

A. Dreismann, et. al. *A sub-femtojoule electrical spinswitch based on optically trapped polariton condensates*, Nat. Mater. 15, 1074 (2016).





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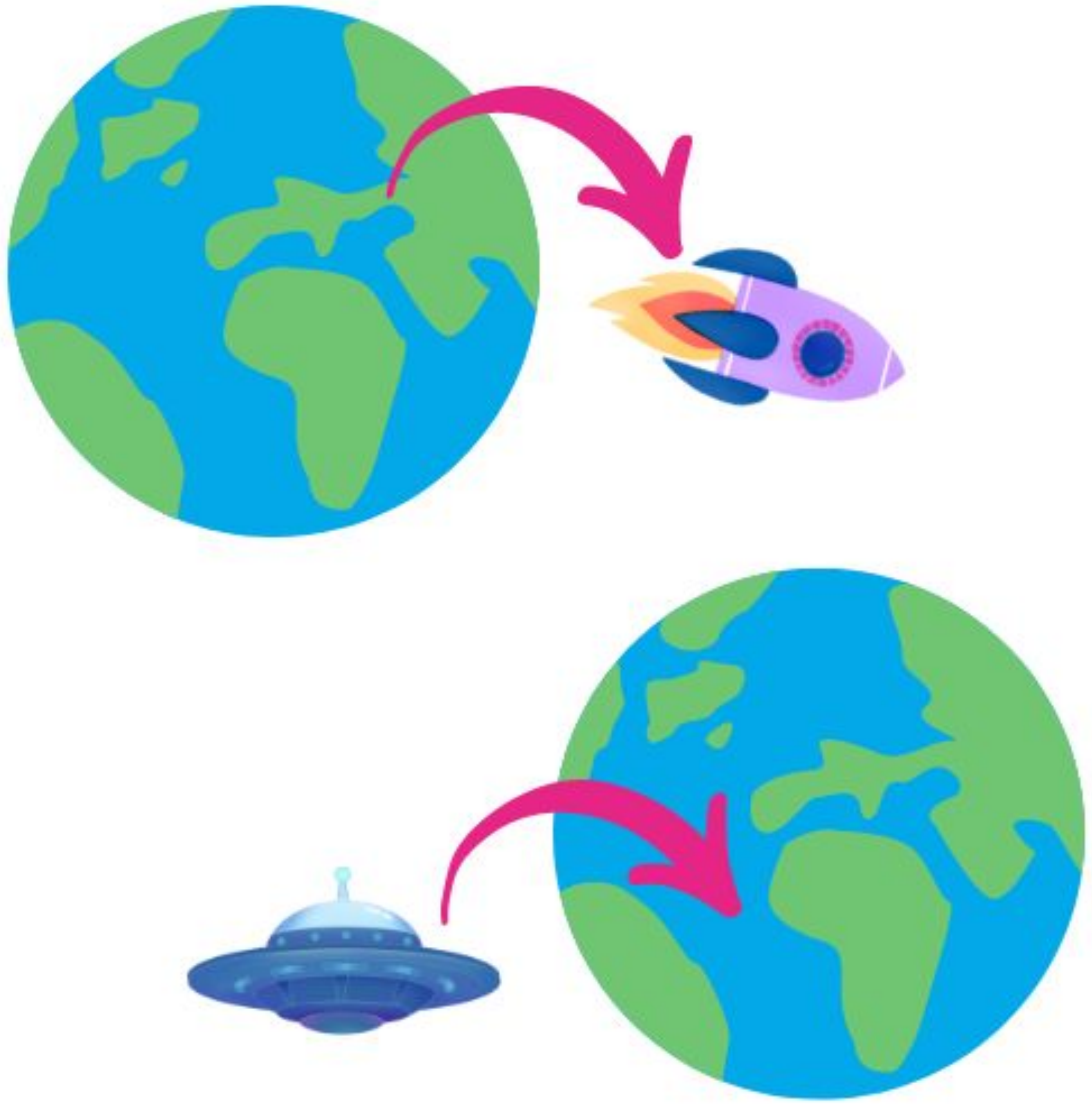
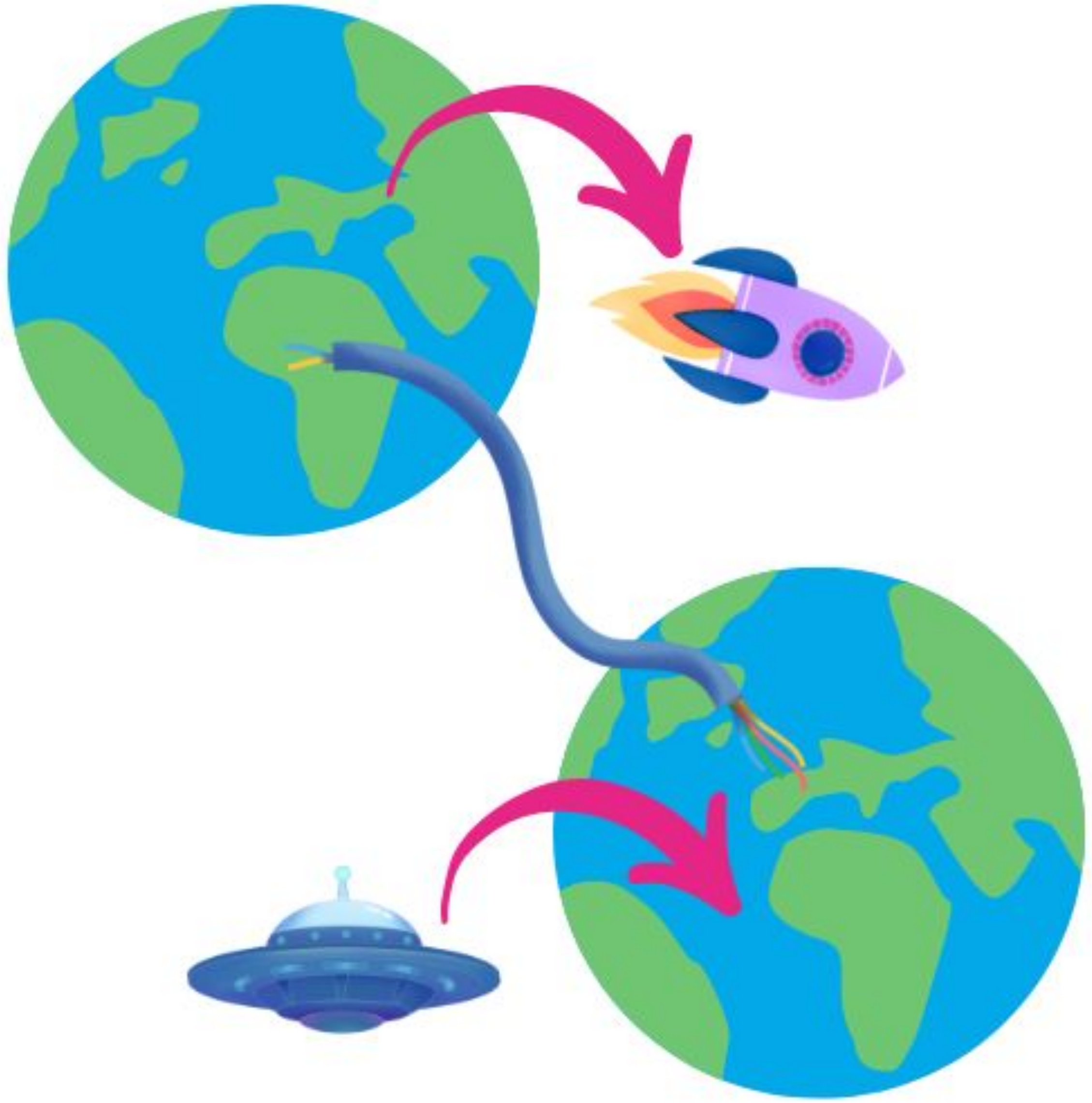
Y. Sekino and L. Susskind, *Fast scramblers*, J. High Energy Phys. 08 (2008) 065.

Tianrui Xu, Thomas Scaffidi, and Xiangyu Cao. Phys. Rev. Lett. **124**, 140602

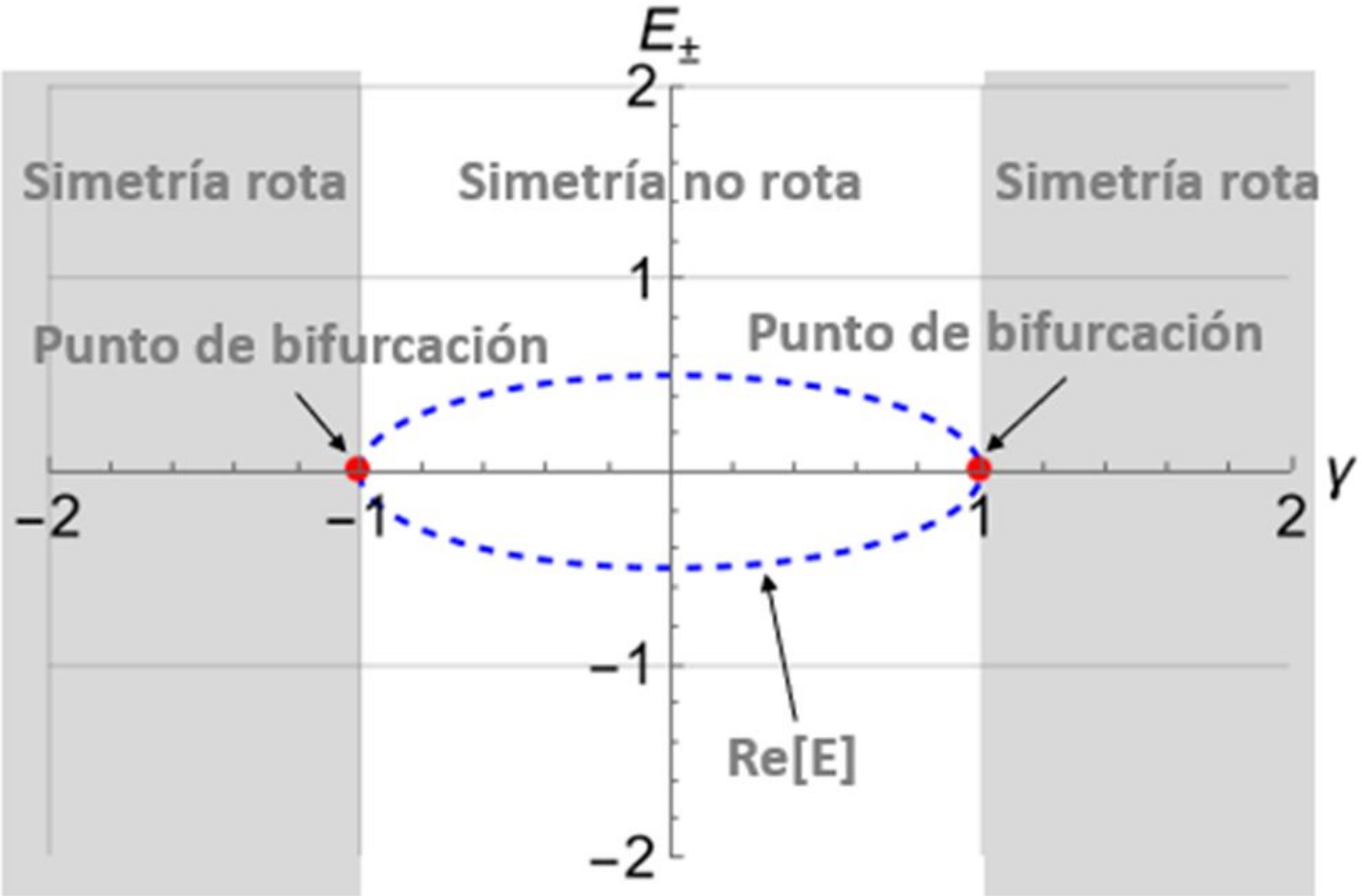
Parity-Time symmetry





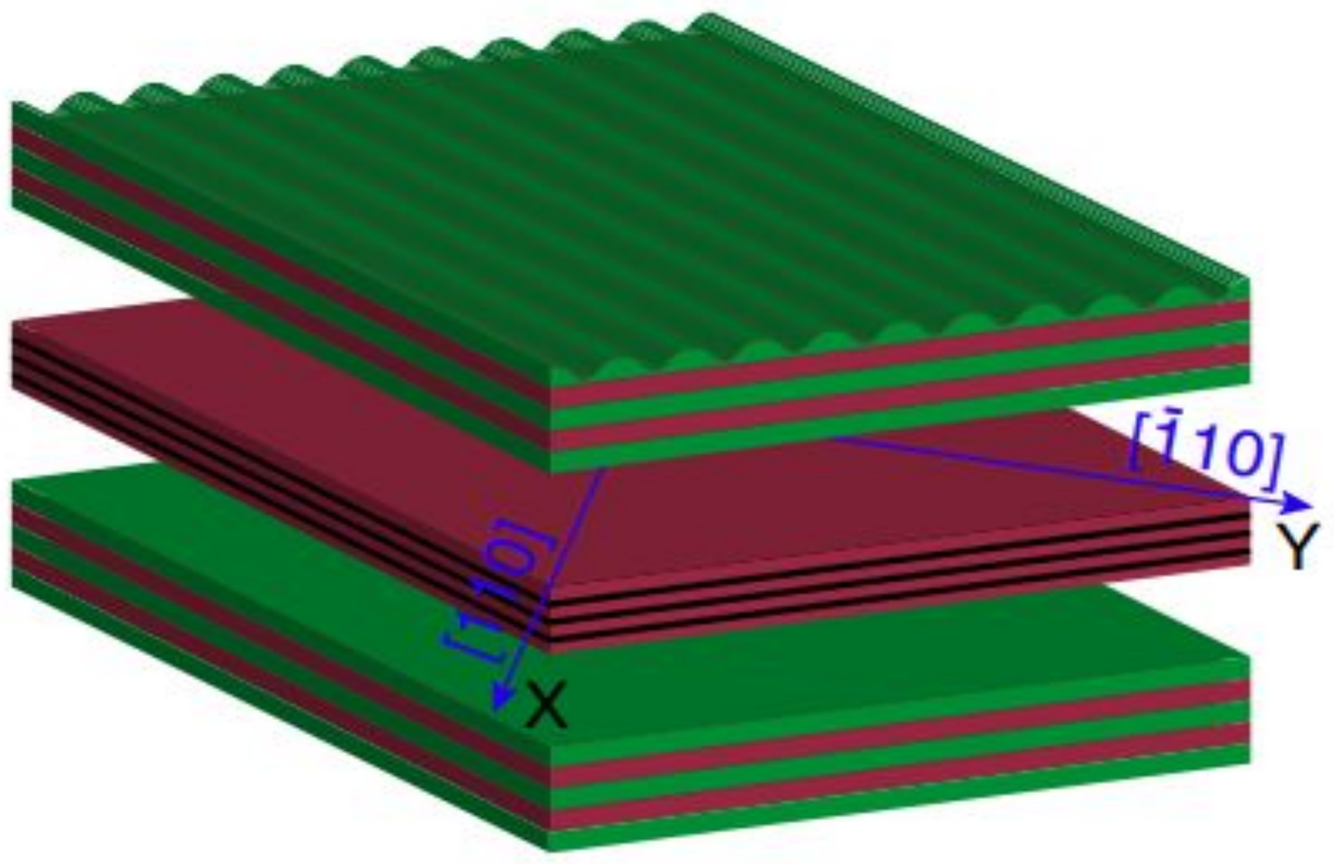
C. M. Bender and S. Boettcher,
Real spectra in non-Hermitian Hamiltonians having PT symmetry,
Phys. Rev. Lett. 80, 5243 (1998).

$$H_a = -\frac{1}{2}\begin{pmatrix} i\gamma & \varepsilon \\ \varepsilon & -i\gamma \end{pmatrix} = -\frac{1}{2}(\varepsilon\sigma_x + i\gamma\sigma_z), \qquad E_\pm = \pm\frac{1}{2}\sqrt{\varepsilon^2 - \gamma^2}$$



02
Model

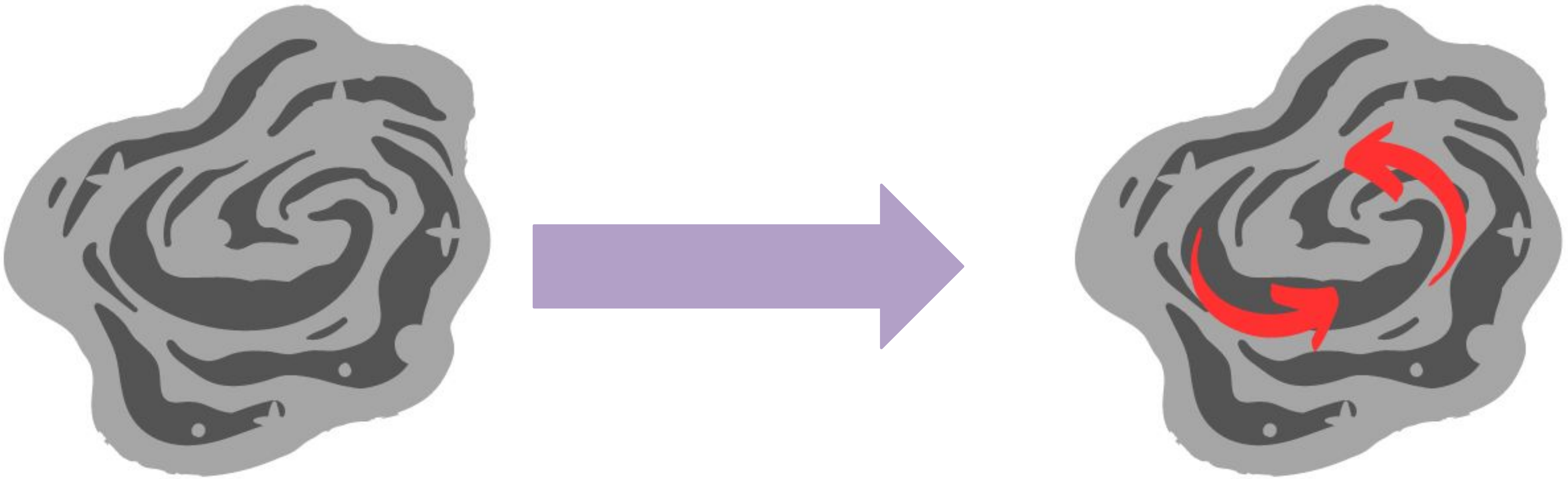
The model



The single polariton Hamiltonian is

$$H = -\frac{1}{2}\begin{pmatrix} ig & \varepsilon - i\varepsilon' + \gamma \\ \varepsilon + i\varepsilon' - \nu & ia \end{pmatrix}, \qquad g = \Gamma - W, \qquad \hat{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{T} = K^*$$

$$\widehat{PT} H \left(\widehat{PT}\right)^{-1} = H$$



Bose-Hubbard model → **Lipkin-Meshkov-Glick model**

The non-Hermitian Bose-Hubbard Hamiltonian is given by

$$\hat{H} = -\frac{(\varepsilon+\gamma)}{2}\hat{\psi}_{+1}^{\dagger}\hat{\psi}_{-1} - \frac{(\varepsilon-\gamma)}{2}\hat{\psi}_{-1}^{\dagger}\hat{\psi}_{+1} + \frac{\alpha}{4}\left(\hat{\psi}_{+1}^{\dagger 2}\hat{\psi}_{+1}^{2} + \hat{\psi}_{-1}^{\dagger 2}\hat{\psi}_{-1}^{2}\right),$$

With the spin operators, the Hamiltonian becomes

$$\hat{s}_k = \frac{1}{2}\left[\Psi^{\dagger}\cdot\sigma_k\cdot\Psi\right], k = 0, x, y, z, \qquad \Psi = \left(\hat{\psi}_{+1}, \hat{\psi}_{-1}\right)$$

$$\hat{\boldsymbol{H}} = \boldsymbol{H}_0(\vec{\hat{\boldsymbol{s}}}) + i\boldsymbol{H}_1(\vec{\hat{\boldsymbol{s}}}),$$

where

$$\boldsymbol{H}_0(\vec{\hat{\boldsymbol{s}}}) = -\boldsymbol{\varepsilon}\hat{\boldsymbol{s}}_x + \frac{\boldsymbol{\alpha}}{2}\left(\hat{\boldsymbol{s}}_z + \hat{\boldsymbol{s}}_0^2 - \hat{\boldsymbol{s}}_0\right), \qquad \boldsymbol{H}_1(\vec{\hat{\boldsymbol{s}}}) = -\boldsymbol{\gamma}\hat{\boldsymbol{s}}_y.$$

Semiclassical approximation

In the mean-field approximation,

$$\frac{d\vec{S}}{dt} = \left[\frac{dH_0}{d\vec{S}} \times \vec{S}\right] + S\,\frac{dH_1}{d\vec{S}},$$

Or in components

$$\dot{S}_x = -\alpha S_z S_y, \qquad \dot{S}_y = -\gamma S + \varepsilon S_z + \alpha S_x S_z, \qquad \dot{S}_z = -\varepsilon S_y, \qquad \dot{S} = -\gamma S_y.$$

Assuming positive values for the parameters $\boldsymbol{\alpha}, \boldsymbol{\varepsilon}, \boldsymbol{\gamma}$ and choosing $\boldsymbol{\varepsilon^{-1}}$ the unit of time we can always set $\varepsilon = 1$ and rescale $\alpha\vec{S} = \vec{s} = \{x, y, z\}$ to obtain

$$\dot{x} = -zy, \qquad \dot{y} = -\gamma s + z + zx, \qquad \dot{z} = -y, \qquad \dot{s} = -\gamma y.$$

We have two integrals of motion and the spin length

$$E = -x + \frac{1}{2}z^2, \qquad \rho = s - \gamma z, \qquad s^2 = x^2 + y^2 + z^2.$$

$$\left(\frac{dz}{dt}\right)^2 = y^2 = s^2 - x^2 - z^2$$

$$\left(\frac{dz}{dt}\right)^2 = (\rho + \gamma z)^2 - \left(\frac{z^2}{2} - E\right)^2 - z^2 = -\frac{1}{4}(z - z_1)(z - z_2)(z - z_3)(z - z_4).$$

$$\boldsymbol{z(t) = \frac{\omega_1 z_4[1 \pm cn(\omega t, m)] + \omega_4 z_1[1 \mp cn(\omega t, m)]}{\omega_1[1 \pm cn(\omega t, m)] + \omega_4[1 \mp cn(\omega t, m)]}},$$

$$\boldsymbol{y(t) = \pm\frac{8\omega^3(z_4 - z_1)dn(\omega t, m)sn(\omega t, m)}{(\omega_1[1 \pm cn(\omega t, m)] + \omega_4[1 \mp cn(\omega t, m)])^2}}$$

$$\boldsymbol{x(t) = \frac{1}{2}z^2(t) - E}$$

Where

$$\omega_1 = \sqrt{(z_1 - z_2)(z_1 - z_3)}, \qquad \omega_4 = \sqrt{(z_3 - z_4)(z_2 - z_4)},$$

$$\omega = \frac{1}{2}\sqrt{\omega_1 \omega_4}, \qquad m = \frac{[\omega_1\omega_4 + (z_1 - z_2)(z_3 - z_4)]^2}{4\omega_1\omega_4(z_1 - z_2)(z_3 - z_4)}.$$

Fixed points

$$\dot{x} = -zy = 0, \qquad \dot{y} = -\gamma s + z + zx = 0, \qquad \dot{z} = -y = 0, \qquad \dot{s} = -\gamma y = 0.$$

$$y = 0, \qquad z = \frac{\text{[illegible]}}{1 - \gamma^2 + x}, \qquad (1 - \gamma^2 + x)^2 x^2 - \rho^2[(1 + x)^2 - \gamma^2] = 0$$

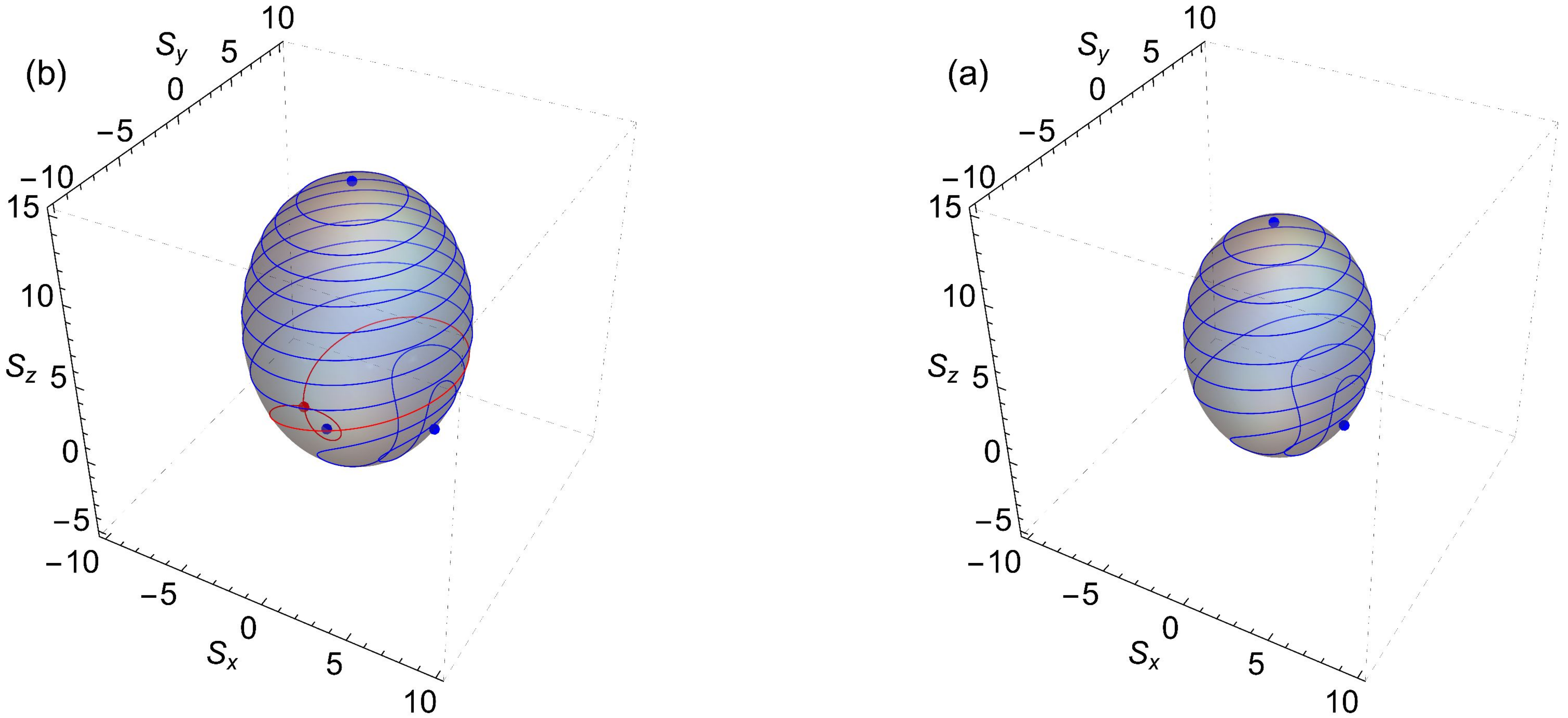
The **determinant of the quartic equation for** x gives us the condition

$$\rho_c = \left[1 + \gamma^{\frac{2}{3}} + \gamma^{\frac{4}{3}}\right]^{3/2}.$$

If $\boldsymbol{\gamma = \gamma/\varepsilon = 1}$ (critical dissipation imbalance) we have $\boldsymbol{\rho_c = \sqrt{27}}$.

Weak dissipation imbalance

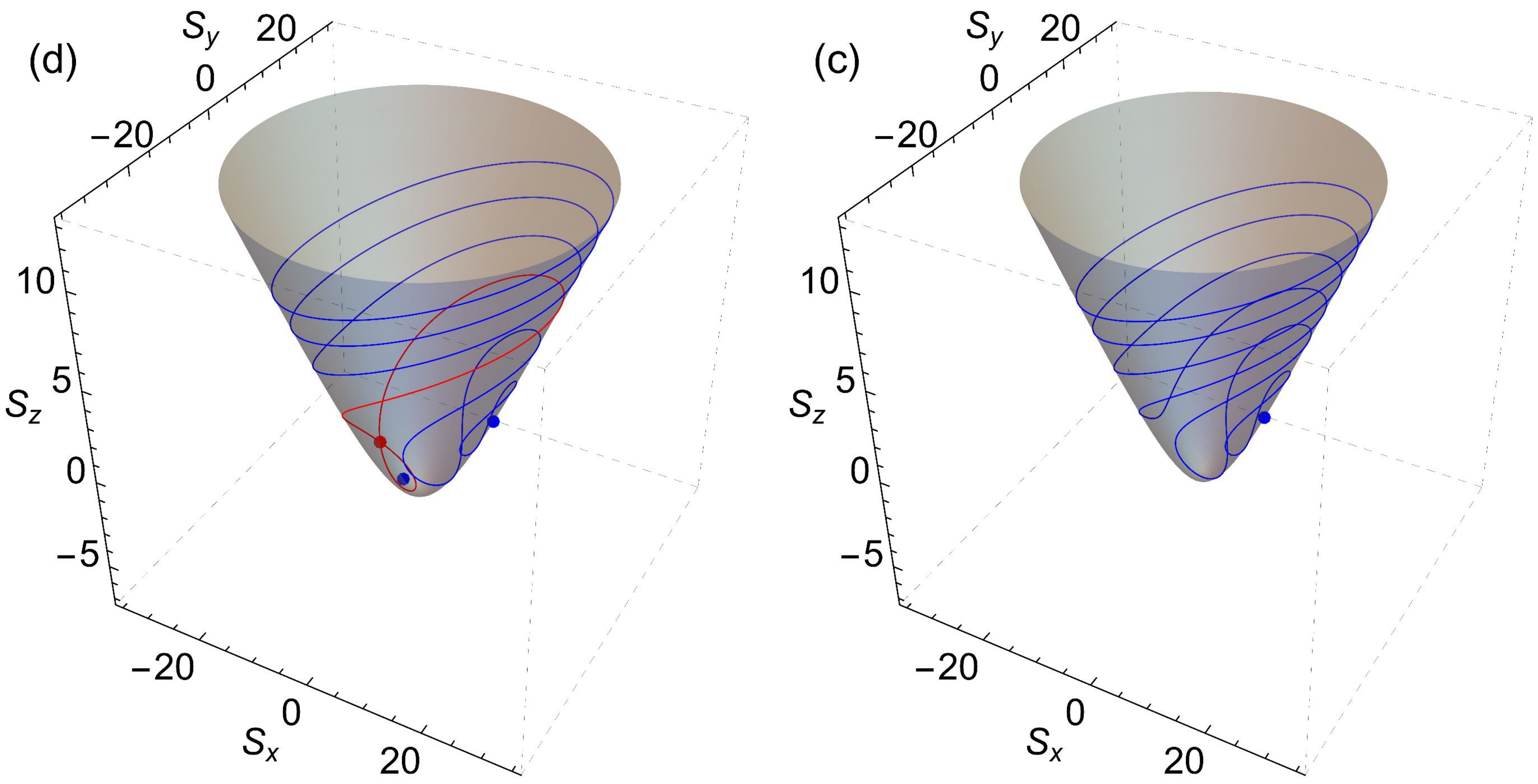
$$\gamma/\varepsilon < 1: \begin{cases} 0 < \rho < \rho_c, & two\ focus \\ \rho > \rho_c, & three\ focus\ and\ one\ saddle \end{cases}$$



$$\gamma/\varepsilon = 0.7\ , \rho_c = 3.741: \quad \rho = 3.6\ (right)\ and\ \rho = 4.3\ (left)$$

Strong dissipation imbalance

$$\gamma/\varepsilon > 1: \begin{cases} 0 < \rho < \rho_c, & one\ focus \\ \rho > \rho_c, & two\ focus\ and\ one\ saddle \end{cases}$$



$$\gamma/\varepsilon = 1.6\ , \rho_c = 8.79: \quad \rho = 8.5\ (right)\ and\ \rho = 10.7\ (left)$$

03

Conclusions

Remarks

- We have shown the **PT-symmetry is unbroken** in our system with the semiclassical approximation.
- The dynamics of the polariton condensate is characterized by **closed pseudoconservative trajectories**.
- We have **analyzed the bifurcations of the fixed points and the topology** of the system
  - **Weak** dissipation imbalance**: ellipsoids**
  - **Critic** dissipation imbalance**: paraboloids**
  - **Strong** dissipation imbalance**: hyperboloids**
  - The fixed points are **focus and saddle**

Thanks for the attention

vereij@ier.unam.mx